

Modélisation de la stratification thermique d'un petit lac urbain

Modèles hybrides alliant physique & *deep learning*

David Métivier (a), Céline Casenave (a) et Brigitte Vinçon-Leite (b)

(a) UMR MISTEA, INRAE

(b) LEESU, École nationale des ponts et chaussées

This talk

Goal: Time series forecasting

Application: Lake temperature and water quality forecasting: here water stratification

Physics based models

- Partial/Ordinary Differential Equations (PDEs/ODEs)
- Expert models

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- Random Forest, MLP
- Auto-regressive models
- Deep learning: RNN, LSTM, Transformer, etc.

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→ **Hybrid models:** Physics-informed + data-driven

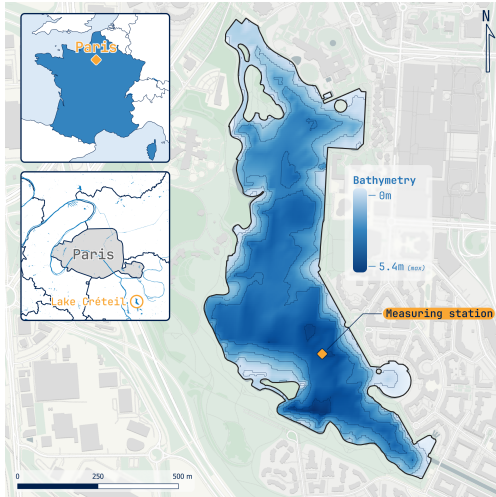


Observil

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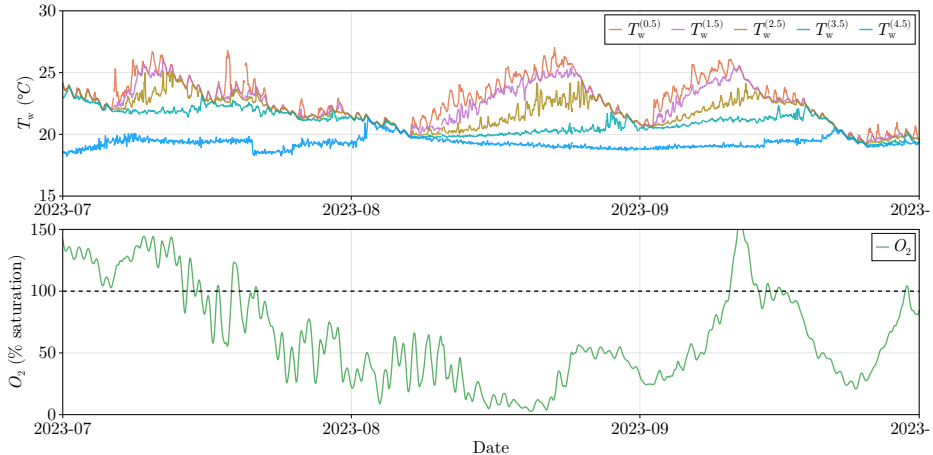
OPUR



Lake Créteil (Credit A. Guillot-Le Goff, 2026)



Lake stratification and oxygen depletion



Measurement frequency: 15 min

- Temperature at 5 depths (0.5, 1.5, 2.5, 3.5, 4.5) m
- Oxygen and Chlorophyll at 1.5 m

Objective

- Easy to deploy hybrid IA-Physics model for lake temperature at multiple depths



Algal bloom in Lake Créteil.

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 - + Other water quality variables (chlorophyll, oxygen, etc.)
 - ++ Anticipating algal blooms
- Oxygen depletion in the lake



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Factors promoting algal blooms

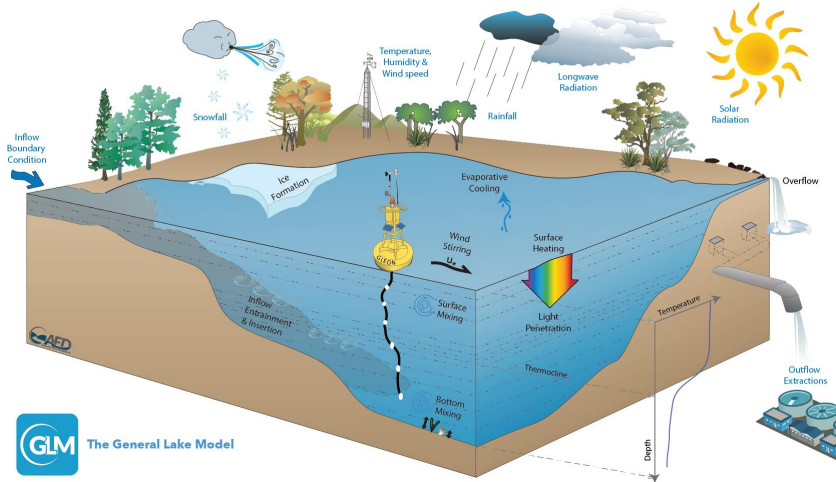
- Temperature increase
- Lake stratification $\rightarrow \Delta T_w = T_w^{(\text{surface})} - T_w^{(\text{bottom})}$
- Increase in nutrients (nitrogen and phosphorus) in the lake $\rightarrow$ not modeled for now



Algal bloom in Lake Créteil.

State of the art: General Lake Model (GLM)

1D hydrodynamical model, based on differential equations



Pros & Cons

- + Interpretable and physics based
- Hard to calibrate
- Require a lot of inputs **hourly**
 - 1 air temperature
 - 2 shortwave radiation
 - 3 longwave radiation
 - 4 relative humidity
 - 5 wind speed
 - 6 precipitation
 - + lake bathymetry
 - + Hydrological forcing (inflow/outflow)
- Hard to couple GLM and Aquatic Eco-Dynamics (AED) model for water quality prediction

State of the art: Data-driven

Different types of models (Random forest, MLP, RNN, LSTM, etc.)

- + Learn with different sources of training data
- + Can learn very complex dynamics
- + Benefit from AI boom
- Time series with seasonal patterns, rare events, changing environment
- Do not know physics
- Sensitive to quantity and quality of data
- Interpretability

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- ⚠ Not a lot of work for multiple temperatures models
- Not state of the art for lake temperature forecast



Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Research papers

Forecasting surface water temperature in lakes: A comparison of approaches

Senlin Zhu^{a,b,*}, Mariusz Ptak^c, Zaher Mundher Yaseen^d, Jiangyu Dai^{a,*}, Bellie Sivakumar^e

^a State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Nanjing Hydraulic Research Institute, Nanjing 210029, China

^b Department of Civil and Environmental Engineering, Cullen College of Engineering, University of Houston, Houston 77204, USA

State of the art: Hybrid models

Neural Networks + physics models

- + Best of both worlds
- + More robust and need less data
- + Satisfy better physics
- ? Can improve reduced models to make them quantitative
- ? Time series with seasonal patterns, rare events, changing environment
- ± Interpretability
 - **How?** → Under active research
 - Harder to train

Universal Differential Equations (Neural ODEs)

For a variable of interest $u(t)$ and an external forcing $g(t)$ we can write the following ODE system,

$$\frac{du}{dt}(t) = \underbrace{f_{\phi}^{(\text{PHY})}(u(t), g(t))}_{\text{known physics}} + \underbrace{f_{\theta}^{(\text{NN})}(u(t), g(t))}_{\text{Neural Network}} .$$

- + Already good with simple NN
- + Physics structure is there to guide the learning
- + Adapted for dynamical systems and forecasting
- Training

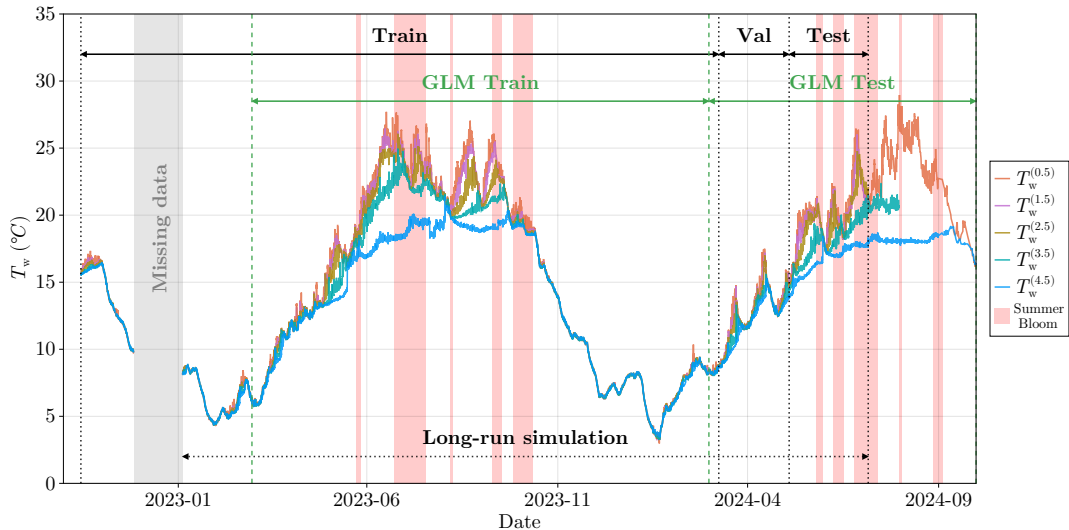
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- Physical term is $f_{\phi}^{(\text{MinPhy})}(T_w(t), T_{\text{air}}(t))$ a model combining a 4-parameter surface model (Air2Temp Piccolroaz et al., 2013) and a simple 8-parameter advection-diffusion model.
- Without the physical, the Neural Network has to learn the whole dynamics $f_{\theta}^{(\text{NODE})}(T_w(t), T_{\text{air}}(t))$

Training/Validation/Test split



Water temperature at different depths.

Objective function

- 1 7-day horizon: operationally relevant (reliable air-temperature forecast window)
- 2 **After** : long-run evaluation (> 18 months) without re-training.

Useful for climate projections

Train on N_W non-overlapping **7-day windows**, all $d_u = 5$ depths jointly:

$$L(\theta) = \overline{\text{RMSE}}(\theta) = \frac{1}{N_W} \sum_{i=1}^{N_W} \sqrt{\frac{1}{d_u n_i} \sum_{z,j} (\hat{U}_{z,j}^{(i)}(\theta) - U_{z,j}^{(i)})^2}$$

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Training details

- Test different activation functions: **Snake**, ReLU, Tanh
- Growing intervals, learning Adam optimizer, early stopping,
- Brute force NN architectures: number of layers, hidden size

7-day forecast: model comparison

Model	N_θ	Train RMSE [$^{\circ}\text{C}$]	Val RMSE [$^{\circ}\text{C}$]	Test RMSE [$^{\circ}\text{C}$]
UDE	586	0.42	0.47	0.55
GLM	~ 20	0.42	0.49	0.61
NODE	595	0.55	0.49	0.71
LSTM	16 625	0.34	0.98	0.75
MinPhy	12	0.96	0.86	1.09

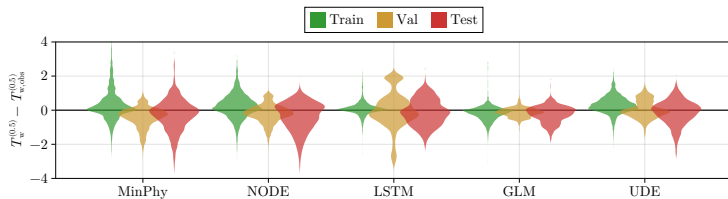
- **UDE**: best val & test RMSE with only 586 parameters ($30\times$ fewer than LSTM)
- **LSTM** overfits: best train RMSE but worst val RMSE (0.98°C)

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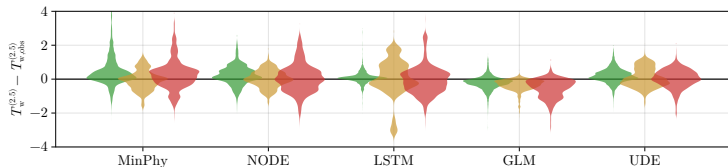
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- **LSTM** overfits: best train RMSE but worst val RMSE (0.98°C)
- UDE excels at depth: RMSE 0.29°C at $z = 4.5\text{ m}$ — best of all models

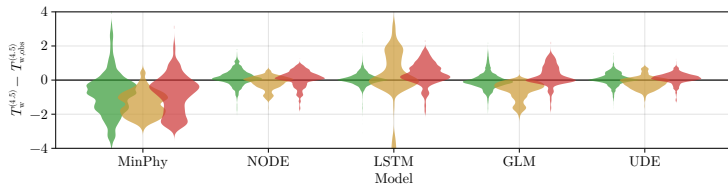
7-day forecast: per-depth errors



Model	Train	Val	Test
MinPhy	0.99	0.73	0.95
NODE	0.75	0.58	0.93
LSTM	0.43	1.06	0.76
GLM	0.5	0.31	0.54
UDE	0.59	0.52	0.72



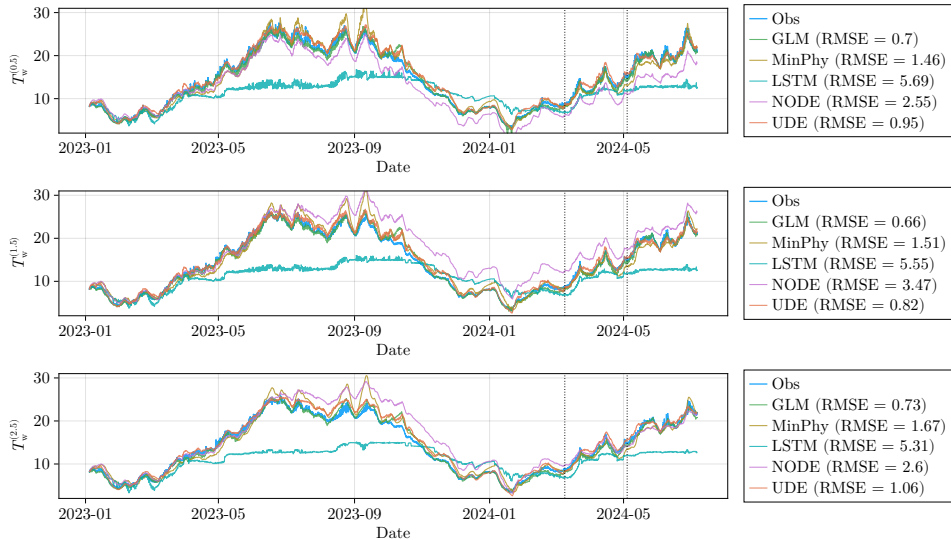
Model	Train	Val	Test
MinPhy	1.11	0.58	0.87
NODE	0.61	0.5	0.83
LSTM	0.37	1.1	0.81
GLM	0.45	0.49	0.78
UDE	0.46	0.56	0.54



Model	Train	Val	Test
MinPhy	1.46	1.55	1.37
NODE	0.45	0.41	0.46
LSTM	0.37	1.52	0.74
GLM	0.46	0.81	0.62
UDE	0.35	0.39	0.32

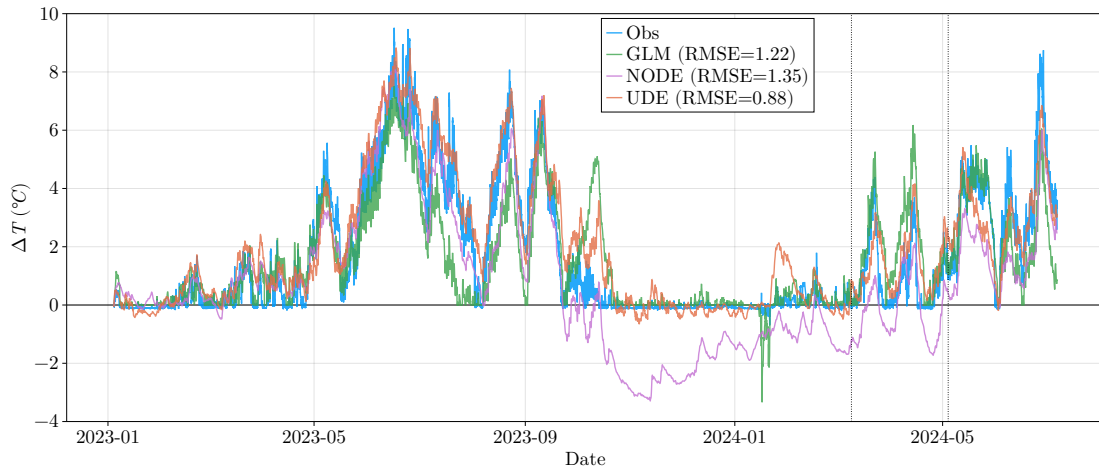
Single long-term prediction (> 18 months)

Long-run RMSE — GLM: 0.78 | MinPhy: 1.65 | LSTM: 5.21 | NODE: 2.64 | UDE: 0.91



Stratification

Stratification $\Delta T = T_w^{(0.5)} - T_w^{(4.5)}$: key indicator for oxygen depletion.



Thermal stratification ΔT over the full period. Dotted lines: train/val/test transitions.

Long-run performance summary

Model	Test 7d RMSE [$^{\circ}\text{C}$]	Long RMSE [$^{\circ}\text{C}$]	Strat ΔT RMSE [$^{\circ}\text{C}$]
UDE	0.55	0.91	0.88
GLM	0.61	0.78	1.22
NODE	0.71	2.64	1.35
LSTM	0.75	5.21	1.78
MinPhy	1.09	1.65	1.59

- **UDE**: best stratification RMSE — despite higher surface error than GLM
- GLM: best long-run RMSE but **requires** full meteo forcings + “manual” calibration
- **NODE & LSTM diverge** on long horizons — no physics to stabilize extrapolation

Conclusions

- UDEs embed physics in the structure of the model and learn the missing physics with a Neural Network.
 - UDEs can learn lake temperature dynamics with minimal input producing "GLM level" results for short and long term forecasting while being far **easier to deploy** (only air temperature as input).
 - It learns the dynamics of stratification ΔT better than any other model.
- **(Ongoing)** Encouraging results for chlorophyll or oxygen concentration but more work needed to get both at the same time.
- Many extensions/improvement possible (e.g. uncertainty quantification, hidden variables, etc.)

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Thank You!